

1 a

$$\begin{aligned}\cot \frac{3\pi}{4} &= \frac{\cos \frac{3\pi}{4}}{\sin \frac{3\pi}{4}} \\ &= -\frac{1}{\sqrt{2}} \div \frac{1}{\sqrt{2}} \\ &= -1\end{aligned}$$

b

$$\begin{aligned}\operatorname{cosec} \frac{5\pi}{4} &= \frac{1}{\sin \frac{5\pi}{4}} \\ &= -\frac{1}{\frac{1}{\sqrt{2}}} \\ &= -\sqrt{2}\end{aligned}$$

c

$$\begin{aligned}\sec \frac{5\pi}{6} &= \frac{1}{\cos \frac{5\pi}{6}} \\ &= \frac{1}{\frac{-\sqrt{3}}{2}} \\ &= -\frac{2}{\sqrt{3}} = -\frac{2\sqrt{3}}{3}\end{aligned}$$

d

$$\begin{aligned}\operatorname{cosec} \frac{\pi}{2} &= \frac{1}{\sin \frac{\pi}{2}} \\ &= \frac{1}{1} = 1\end{aligned}$$

e

$$\begin{aligned}\sec \frac{4\pi}{3} &= \frac{1}{\cos \frac{4\pi}{3}} \\ &= \frac{1}{-\frac{1}{2}} = -2\end{aligned}$$

f

$$\begin{aligned}\operatorname{cosec} \frac{13\pi}{6} &= \frac{1}{\sin \frac{13\pi}{6}} \\ &= \frac{1}{\sin \frac{\pi}{6}} \\ &= \frac{1}{\frac{1}{2}} = 2\end{aligned}$$

g

$$\begin{aligned}\cot \frac{7\pi}{3} &= \frac{\cos \frac{7\pi}{3}}{\sin \frac{7\pi}{3}} \\ &= \frac{1}{2} \div \frac{\sqrt{3}}{2} \\ &= \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}\end{aligned}$$

$$\begin{aligned} \text{h} \quad \sec \frac{5\pi}{3} &= \frac{1}{\cos \frac{5\pi}{3}} \\ &= \frac{1}{\frac{1}{2}} = 2 \end{aligned}$$

$$\begin{aligned} 2 \quad \text{a} \quad \cot 135^\circ &= \frac{\cos 135^\circ}{\sin 135^\circ} \\ &= -\frac{1}{\sqrt{2}} \div \frac{1}{\sqrt{2}} \\ &= -1 \end{aligned}$$

$$\begin{aligned} \text{b} \quad \sec 150^\circ &= \frac{1}{\cos 150^\circ} \\ &= \frac{1}{-\frac{\sqrt{3}}{2}} \\ &= -\frac{2}{\sqrt{3}} = -\frac{2\sqrt{3}}{3} \end{aligned}$$

$$\begin{aligned} \text{c} \quad \operatorname{cosec} 90^\circ &= \frac{1}{\sin 90^\circ} \\ &= \frac{1}{1} = 1 \end{aligned}$$

$$\begin{aligned} \text{d} \quad \cot 240^\circ &= \frac{\cos 240^\circ}{\sin 240^\circ} \\ &= -\frac{1}{2} \div -\frac{\sqrt{3}}{2} \\ &= \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3} \end{aligned}$$

$$\begin{aligned} \text{e} \quad \sec 225^\circ &= \frac{1}{\cos 225^\circ} \\ &= \frac{1}{-\frac{1}{\sqrt{2}}} \\ &= -\sqrt{2} \end{aligned}$$

$$\begin{aligned} \text{f} \quad \sec 330^\circ &= \frac{1}{\cos 330^\circ} \\ &= \frac{1}{\frac{\sqrt{3}}{2}} \\ &= \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3} \end{aligned}$$

$$\begin{aligned} \text{g} \quad \cot 315^\circ &= \frac{\cos 315^\circ}{\sin 315^\circ} \\ &= \frac{1}{\sqrt{2}} \div -\frac{1}{\sqrt{2}} \\ &= -1 \end{aligned}$$

$$\begin{aligned}
 \text{h} \quad \operatorname{cosec} 300^\circ &= \frac{1}{\sin 300^\circ} \\
 &= \frac{1}{-\frac{\sqrt{3}}{2}} \\
 &= -\frac{2}{\sqrt{3}} = -\frac{2\sqrt{3}}{3}
 \end{aligned}$$

$$\begin{aligned}
 \text{i} \quad \cot 420^\circ &= \frac{\cos 420^\circ}{\sin 420^\circ} \\
 &= \frac{\cos 60^\circ}{\sin 60^\circ} \\
 &= \frac{1}{2} \div \frac{\sqrt{3}}{2} \\
 &= \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}
 \end{aligned}$$

$$\begin{aligned}
 \text{3 a} \quad \operatorname{cosec} x &= 2 \\
 \sin x &= \frac{1}{2} \\
 x &= \frac{\pi}{6}, \frac{5\pi}{6}
 \end{aligned}$$

$$\begin{aligned}
 \text{b} \quad \cot x &= \sqrt{3} \\
 \tan x &= \frac{1}{\sqrt{3}} \\
 x &= \frac{\pi}{6}, \frac{7\pi}{6}
 \end{aligned}$$

$$\begin{aligned}
 \text{c} \quad \sec x &= -\sqrt{2} \\
 \cos x &= -\frac{1}{\sqrt{2}} \\
 x &= \frac{3\pi}{4}, \frac{5\pi}{4}
 \end{aligned}$$

$$\begin{aligned}
 \text{d} \quad \operatorname{cosec} x &= \sec x \\
 \sin x &= \cos x \\
 \tan x &= 1 \\
 x &= \frac{\pi}{4}, \frac{5\pi}{4}
 \end{aligned}$$

$$\begin{aligned}
 \text{4 a} \quad \cos \theta &= \frac{1}{\frac{\sec \theta}{8}} \\
 &= -\frac{8}{17}
 \end{aligned}$$

$$\begin{aligned}
 \text{b} \quad \cos^2 \theta + \sin^2 \theta &= 1 \\
 \frac{64}{289} + \sin^2 \theta &= 1 \\
 \sin^2 \theta &= \frac{225}{289} \\
 \sin \theta &= \frac{15}{17} \quad (\text{Since } \sin \theta > 0)
 \end{aligned}$$

$$\begin{aligned} \text{c } \tan \theta &= \frac{\sin \theta}{\cos \theta} \\ &= \frac{15}{17} \div -\frac{8}{17} \\ &= -\frac{15}{8} \end{aligned}$$

$$\begin{aligned} 5 \quad 1 + \tan^2 \theta &= \sec^2 \theta \\ \sec^2 \theta &= 1 + \frac{49}{576} = \frac{625}{576} \\ \sec \theta &= \frac{25}{24} \quad (\text{since } \cos \theta > 0) \\ \cos \theta &= \frac{24}{25} \\ \frac{\sin \theta}{\cos \theta} &= \tan \theta = -\frac{7}{24} \\ \sin \theta &= -\frac{7}{24} \times \frac{24}{25} \\ &= -\frac{7}{25} \end{aligned}$$

$$\begin{aligned} 6 \quad 1 + \tan^2 \theta &= \sec^2 \theta \\ \sec^2 \theta &= 1 + 0.16 = 1.16 \\ \sec \theta &= -\sqrt{\frac{116}{100}} \\ (\text{Since } \theta \text{ is in the 3rd quadrant}) \\ &= -\sqrt{\frac{29}{25}} \\ &= -\frac{\sqrt{29}}{5} \end{aligned}$$

$$\begin{aligned} 7 \quad \cot \theta &= \frac{3}{4} \\ 1 + \tan^2 \theta &= \sec^2 \theta \\ \sec^2 \theta &= 1 + \frac{16}{9} = \frac{25}{9} \\ \sec \theta &= -\frac{5}{3} \quad (\cos \theta < 0) \\ \cos \theta &= -\frac{3}{5} \\ \frac{\sin \theta}{\cos \theta} &= \tan \theta = \frac{4}{3} \\ \sin \theta &= \frac{4}{3} \times -\frac{3}{5} \\ &= -\frac{4}{5} \\ \frac{\sin \theta - 2 \cos \theta}{\cot \theta - \sin \theta} &= \frac{-\frac{4}{5} - -\frac{6}{5}}{\frac{3}{4} - -\frac{4}{5}} \\ &= \frac{2}{5} \div \frac{31}{20} \\ &= \frac{2}{5} \times \frac{20}{31} = \frac{8}{31} \end{aligned}$$

$$8 \quad \cos^2 \theta + \sin^2 \theta = 1$$

$$\frac{4}{9} + \sin^2 \theta = 1$$

$$\sin^2 \theta = \frac{5}{9}$$

$$\sin \theta = -\frac{\sqrt{5}}{3} \left(\frac{3\pi}{2} < \theta < 2\pi \right)$$

$$\tan \theta = -\frac{\sqrt{5}}{3} \div \frac{2}{3} = -\frac{\sqrt{5}}{2}$$

$$\cot \theta = -\frac{2}{\sqrt{5}}$$

$$\begin{aligned} \frac{\tan \theta - 3 \sin \theta}{\cos \theta - 2 \cot \theta} &= \frac{-\frac{\sqrt{5}}{2} - \left(-\sqrt{5}\right)}{\frac{2}{3} - \left(-\frac{4}{\sqrt{5}}\right)} \\ &= \frac{\sqrt{5}}{2} \div \frac{2\sqrt{5} + 12}{3\sqrt{5}} \\ &= \frac{\sqrt{5}}{2} \times \frac{3\sqrt{5}}{2\sqrt{5} + 12} \\ &= \frac{15}{4(\sqrt{5} + 6)} \times \frac{6 - \sqrt{5}}{6 - \sqrt{5}} \\ &= \frac{15(6 - \sqrt{5})}{4 \times (36 - 5)} \\ &= \frac{15(6 - \sqrt{5})}{124} \end{aligned}$$

$$\begin{aligned} 9 \text{ a} \quad (1 - \cos^2 \theta)(1 + \cot^2 \theta) &= \sin^2 \theta \times \cot^2 \theta \\ &= \sin^2 \theta \times \frac{\cos^2 \theta}{\sin^2 \theta} \\ &= \cos^2 \theta, \text{ provided } \sin \theta \neq 0 \end{aligned}$$

If $\sin \theta = 0$, $\cot \theta$ would be undefined.

$$\begin{aligned} \text{b} \quad \cos^2 \theta \tan^2 \theta + \sin^2 \theta \cot^2 \theta &= \cos^2 \theta \times \frac{\sin^2 \theta}{\cos^2 \theta} + \sin^2 \theta \times \frac{\cos^2 \theta}{\sin^2 \theta} \\ &= \sin^2 \theta + \cos^2 \theta \\ &= 1, \text{ provided } \sin \theta \neq 0 \text{ and } \cos \theta \neq 0 \end{aligned}$$

c In cases like this, it is a good strategy to start with the more complicated expression.

$$\begin{aligned} \frac{\tan \theta + \cot \phi}{\cot \theta + \tan \phi} &= \frac{\frac{\sin \theta}{\cos \theta} + \frac{\cos \phi}{\sin \phi}}{\frac{\cos \theta}{\sin \theta} + \frac{\sin \phi}{\cos \phi}} \\ &= \frac{\frac{\sin \theta \sin \phi + \cos \phi \cos \theta}{\cos \theta \sin \phi}}{\frac{\cos \phi \cos \theta + \sin \theta \cos \phi}{\cos \phi \sin \theta}} \\ &= \frac{\sin \theta \sin \phi + \cos \phi \cos \theta}{\cos \theta \sin \phi} \times \frac{\cos \phi \sin \theta}{\cos \phi \cos \theta + \sin \theta \cos \phi} \\ \frac{\tan \theta + \cot \phi}{\cot \theta + \tan \phi} &= \frac{\cos \phi \sin \theta}{\cos \theta \sin \phi} \\ &= \frac{\sin \theta}{\cos \theta} \times \frac{\cos \phi}{\sin \phi} \\ &= \frac{\sin \theta}{\cos \theta} \div \frac{\sin \phi}{\cos \phi} \\ &= \frac{\tan \theta}{\tan \phi} \end{aligned}$$

This is provided $\cot \theta + \tan \phi \neq 0$ and the tangent and cotangent are defined.

$$\begin{aligned} \mathbf{d} \quad (\sin \theta + \cos \theta)^2 + (\sin \theta - \cos \theta)^2 &= \sin^2 \theta + 2 \sin \theta \cos \theta + \cos^2 \theta \\ &\quad + \sin^2 \theta - 2 \sin \theta \cos \theta + \cos^2 \theta \\ &= 2 \sin^2 \theta + 2 \cos^2 \theta \\ &= 2 \end{aligned}$$

There are no restrictions on θ .

$$\begin{aligned} \mathbf{e} \quad \frac{1 + \cot^2 \theta}{\cot \theta \operatorname{cosec} \theta} &= \frac{\operatorname{cosec}^2 \theta}{\cot \theta \operatorname{cosec} \theta} \\ &= \frac{\operatorname{cosec} \theta}{\cot \theta} \\ &= \frac{1}{\sin \theta} \times \frac{\sin \theta}{\cos \theta} \\ &= \frac{1}{\cos \theta} \\ &= \sec \theta \end{aligned}$$

Conditions: $\sin \theta \neq 0, \cos \theta \neq 0$

$$\begin{aligned} \mathbf{f} \quad \sec \theta + \tan \theta &= \frac{1}{\cos \theta} + \frac{\sin \theta}{\cos \theta} \\ &= \frac{1 + \sin \theta}{\cos \theta} \times \frac{1 - \sin \theta}{1 - \sin \theta} \\ &= \frac{(1 + \sin \theta)(1 - \sin \theta)}{\cos \theta(1 - \sin \theta)} \\ &= \frac{1 - \sin^2 \theta}{\cos \theta(1 - \sin \theta)} \\ &= \frac{\cos^2 \theta}{\cos \theta(1 - \sin \theta)} \\ &= \frac{\cos \theta}{1 - \sin \theta} \end{aligned}$$

Conditions: $\cos \theta \neq 0$ (includes $\sin \theta \neq 1$)