Exercise 11A: Solutions

1 a

$$\cot\frac{3\pi}{4}=\frac{\cos\frac{3\pi}{4}}{\sin\frac{3\pi}{4}}$$
$$=-\frac{1}{\sqrt{2}}\div\frac{1}{\sqrt{2}}$$
$$=-1$$

b

$$\operatorname{cosec}\frac{5\pi}{4}=\frac{1}{\sin\frac{5\pi}{4}}$$
$$=-\frac{1}{\frac{1}{\sqrt{2}}}$$
$$=-\sqrt{2}$$

c

$$\sec\frac{5\pi}{6}=\frac{1}{\cos\frac{5\pi}{6}}$$
$$=\frac{1}{\frac{-\sqrt{3}}{2}}$$
$$=-\frac{2}{\sqrt{3}}=-\frac{2\sqrt{3}}{3}$$

d

$$\operatorname{cosec}\frac{\pi}{2}=\frac{1}{\sin\frac{\pi}{2}}$$
$$=\frac{1}{1}=1$$

e

$$\sec\frac{4\pi}{3}=\frac{1}{\cos\frac{4\pi}{3}}$$
$$=\frac{1}{-\frac{1}{2}}=-2$$

f

$$\operatorname{cosec}\frac{13\pi}{6}=\frac{1}{\sin\frac{13\pi}{6}}$$
$$=\frac{1}{\sin\frac{\pi}{6}}$$
$$=\frac{1}{\frac{1}{2}}=2$$

g

$$\cot\frac{7\pi}{3}=\frac{\cos\frac{7\pi}{3}}{\sin\frac{7\pi}{3}}$$
$$=\frac{1}{2}\div\frac{\sqrt{3}}{2}$$
$$=\frac{1}{\sqrt{3}}=\frac{\sqrt{3}}{3}$$

h $\sec\frac{5\pi}{3} = \frac{1}{\cos\frac{5\pi}{3}}$

$= \frac{1}{\frac{1}{2}} = 2$

2 **a** $\cot 135^\circ = \frac{\cos 135^\circ}{\sin 135^\circ}$

$= -\frac{1}{\sqrt{2}} \div \frac{1}{\sqrt{2}}$

$= -1$

b $\sec 150^\circ = \frac{1}{\cos 150^\circ}$

$= \frac{1}{-\frac{\sqrt{3}}{2}}$

$= -\frac{2}{\sqrt{3}} = -\frac{2\sqrt{3}}{3}$

c $\operatorname{cosec} 90^\circ = \frac{1}{\sin 90^\circ}$

$= \frac{1}{1} = 1$

d $\cot 240^\circ = \frac{\cos 240^\circ}{\sin 240^\circ}$

$= -\frac{1}{2} \div -\frac{\sqrt{3}}{2}$

$= \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

e $\sec 225^\circ = \frac{1}{\cos 225^\circ}$

$= \frac{1}{-\frac{1}{\sqrt{2}}}$

$= -\sqrt{2}$

f $\sec 330^\circ = \frac{1}{\cos 330^\circ}$

$= \frac{1}{\frac{\sqrt{3}}{2}}$

$= \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$

g $\cot 315^\circ = \frac{\cos 315^\circ}{\sin 315^\circ}$

$= \frac{1}{\sqrt{2}} \div -\frac{1}{\sqrt{2}}$

$= -1$

h $\operatorname{cosec} 300^\circ = \dfrac{1}{\sin 300^\circ}$

$= \dfrac{1}{\frac{-\sqrt{3}}{2}}$

$= -\dfrac{2}{\sqrt{3}} = -\dfrac{2\sqrt{3}}{3}$

i $\cot 420^\circ = \dfrac{\cos 420^\circ}{\sin 420^\circ}$

$= \dfrac{\cos 60^\circ}{\sin 60^\circ}$

$= \dfrac{1}{2} \div \dfrac{\sqrt{3}}{2}$

$= \dfrac{1}{\sqrt{3}} = \dfrac{\sqrt{3}}{3}$

3 a $\operatorname{cosec} x = 2$

$\sin x = \dfrac{1}{2}$

$x = \dfrac{\pi}{6}, \dfrac{5\pi}{6}$

b $\cot x = \sqrt{3}$

$\tan x = \dfrac{1}{\sqrt{3}}$

$x = \dfrac{\pi}{6}, \dfrac{7\pi}{6}$

c $\sec x = -\sqrt{2}$

$\cos x = -\dfrac{1}{\sqrt{2}}$

$x = \dfrac{3\pi}{4}, \dfrac{5\pi}{4}$

d $\operatorname{cosec} x = \sec x$

$\sin x = \cos x$

$\tan x = 1$

$x = \dfrac{\pi}{4}, \dfrac{5\pi}{4}$

4 a $\cos\theta = \dfrac{1}{\sec\theta}$

$= -\dfrac{8}{17}$

b $\cos^2\theta + \sin^2\theta = 1$

$\dfrac{64}{289} + \sin^2\theta = 1$

$\sin^2\theta = \dfrac{225}{289}$

$\sin\theta = \dfrac{15}{17}$ (Since $\sin\theta > 0$)

c $\tan\theta = \frac{\sin\theta}{\cos\theta}$

$= \frac{15}{17} \div -\frac{8}{17}$

$= -\frac{15}{8}$

5 $1 + \tan^2\theta = \sec^2\theta$

$\sec^2\theta = 1 + \frac{49}{576} = \frac{625}{576}$

$\sec\theta = \frac{25}{24}$ (since $\cos\theta > 0$)

$\cos\theta = \frac{24}{25}$

$\frac{\sin\theta}{\cos\theta} = \tan\theta = -\frac{7}{24}$

$\sin\theta = -\frac{7}{24} \times \frac{24}{25}$

$= -\frac{7}{25}$

6 $1 + \tan^2\theta = \sec^2\theta$

$\sec^2\theta = 1 + 0.16 = 1.16$

$\sec\theta = -\sqrt{\frac{116}{100}}$

(Since θ is in the 3rd quadrant)

$= -\sqrt{\frac{29}{25}}$

$= -\frac{\sqrt{29}}{5}$

7 $\cot\theta = \frac{3}{4}$

$1 + \tan^2\theta = \sec^2\theta$

$\sec^2\theta = 1 + \frac{16}{9} = \frac{25}{9}$

$\sec\theta = -\frac{5}{3} (\cos\theta < 0)$

$\cos\theta = -\frac{3}{5}$

$\frac{\sin\theta}{\cos\theta} = \tan\theta = \frac{4}{3}$

$\sin\theta = \frac{4}{3} \times -\frac{3}{5}$

$= -\frac{4}{5}$

$\frac{\sin\theta - 2\cos\theta}{\cot\theta - \sin\theta} = \frac{-\frac{4}{5} - -\frac{6}{5}}{\frac{3}{4} - -\frac{4}{5}}$

$= \frac{2}{5} \div \frac{31}{20}$

$= \frac{2}{5} \times \frac{20}{31} = \frac{8}{31}$

8 $\cos^2\theta + \sin^2\theta = 1$

$$\frac{4}{9} + \sin^2\theta = 1$$

$$\sin^2\theta = \frac{5}{9}$$

$$\sin\theta = -\frac{\sqrt{5}}{3}\left(\frac{3\pi}{2} < \theta < 2\pi\right)$$

$$\tan\theta = -\frac{\sqrt{5}}{3} \div \frac{2}{3} = -\frac{\sqrt{5}}{2}$$

$$\cot\theta = -\frac{2}{\sqrt{5}}$$

$$\frac{\tan\theta - 3\sin\theta}{\cos\theta - 2\cot\theta} = \frac{-\frac{\sqrt{5}}{2} - \left(-\sqrt{5}\right)}{\frac{2}{3} - \left(-\frac{4}{\sqrt{5}}\right)}$$

$$= \frac{\sqrt{5}}{2} \div \frac{2\sqrt{5}+12}{3\sqrt{5}}$$

$$= \frac{\sqrt{5}}{2} \times \frac{3\sqrt{5}}{2\sqrt{5}+12}$$

$$= \frac{15}{4(\sqrt{5}+6)} \times \frac{6-\sqrt{5}}{6-\sqrt{5}}$$

$$= \frac{15(6-\sqrt{5})}{4\times(36-5)}$$

$$= \frac{15(6-\sqrt{5})}{124}$$

9 a $(1-\cos^2\theta)(1+\cot^2\theta) = \sin^2\theta \times \cot^2\theta$

$$= \sin^2\theta \times \frac{\cos^2\theta}{\sin^2\theta}$$

$$= \cos^2\theta, \text{provided} \sin\theta \neq 0$$

If $\sin\theta = 0$, $\cot\theta$ would be undefined.

b $\cos^2\theta\ \tan^2\theta + \sin^2\theta\cot 2\theta = \cos^2\theta \times \dfrac{\sin^2\theta}{\cos^2\theta} + \sin^2\theta \times \dfrac{\cos^2\theta}{\sin^2\theta}$

$$= \sin^2\theta + \cos^2\theta$$

$$= 1, \text{provided} \sin\theta \neq 0 \text{ and} \cos\theta \neq 0$$

c In cases like this, it is a good strategy to start with the more complicated expression.

$$\begin{aligned}\frac{\tan\theta+\cot\phi}{\cot\theta+\tan\phi}&=\frac{\frac{\sin\theta}{\cos\theta}+\frac{\cos\phi}{\sin\phi}}{\frac{\cos\theta}{\sin\theta}+\frac{\sin\phi}{\cos\phi}}\\&=\frac{\frac{\sin\theta\sin\phi+\cos\phi\cos\theta}{\cos\theta\sin\phi}}{\frac{\cos\phi\cos\theta+\sin\theta\cos\phi}{\cos\phi\sin\theta}}\\&=\frac{\sin\theta\sin\phi+\cos\phi\cos\theta}{\cos\theta\sin\phi}\\&\quad\times\frac{\cos\phi\sin\theta}{\cos\phi\cos\theta+\sin\theta\cos\phi}\end{aligned}$$

$$\begin{aligned}\frac{\tan\theta+\cot\phi}{\cot\theta+\tan\phi}&=\frac{\cos\phi\sin\theta}{\cos\theta\sin\phi}\\&=\frac{\sin\theta}{\cos\theta}\times\frac{\cos\phi}{\sin\phi}\\&=\frac{\sin\theta}{\cos\theta}\div\frac{\sin\phi}{\cos\phi}\\&=\frac{\tan\theta}{\tan\phi}\end{aligned}$$

This is provided $\cot\theta+\tan\phi\neq0$ and the tangent and cotangent are defined.

d
$$\begin{aligned}(\sin\theta+\cos\theta)^2+(\sin\theta-\cos\theta)^2&=\sin^2\theta+2\sin\theta\cos\theta+\cos^2\theta\\&\quad+\sin^2\theta-2\sin\theta\cos\theta+\cos^2\theta\\&=2\sin^2\theta+2\cos^2\theta\\&=2\end{aligned}$$

There are no restrictions on θ.

e
$$\begin{aligned}\frac{1+\cot^2\theta}{\cot\theta\operatorname{cosec}\theta}&=\frac{\operatorname{cosec}^2\theta}{\cot\theta\operatorname{cosec}\theta}\\&=\frac{\operatorname{cosec}\theta}{\cot\theta}\\&=\frac{1}{\sin\theta}\times\frac{\sin\theta}{\cos\theta}\\&=\frac{1}{\cos\theta}\\&=\sec\theta\end{aligned}$$

Conditions: $\sin\theta\neq0, \cos\theta\neq0$

f
$$\begin{aligned}\sec\theta+\tan\theta&=\frac{1}{\cos\theta}+\frac{\sin\theta}{\cos\theta}\\&=\frac{1+\sin\theta}{\cos\theta}\times\frac{1-\sin\theta}{1-\sin\theta}\\&=\frac{(1+\sin\theta)(1-\sin\theta)}{\cos\theta(1-\sin\theta)}\\&=\frac{1-\sin^2\theta}{\cos\theta(1-\sin\theta)}\\&=\frac{\cos^2\theta}{\cos\theta(1-\sin\theta)}\\&=\frac{\cos\theta}{1-\sin\theta}\end{aligned}$$

Conditions: $\cos\theta\neq0$ (includes $\sin\theta\neq1$)